Vertical shifts

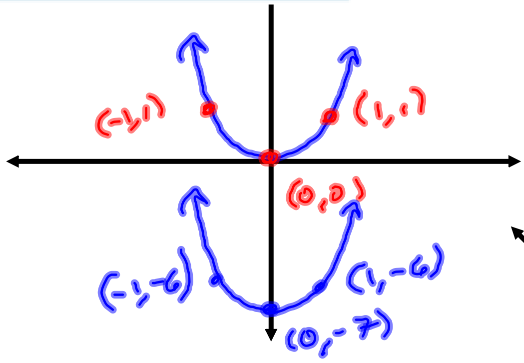
$y = f(x) + k, \quad k > 0$ Raise the graph of f by k units.

$y = f(x) - k, \quad k > 0$ Lower the graph of f by k units.

$f(x) = x^2$

$x^2 - 7$: Down 7

$f(x) - 7$



Horizontal shifts

$y = f(x + h), \quad h > 0$ Left shift, Advance the spark

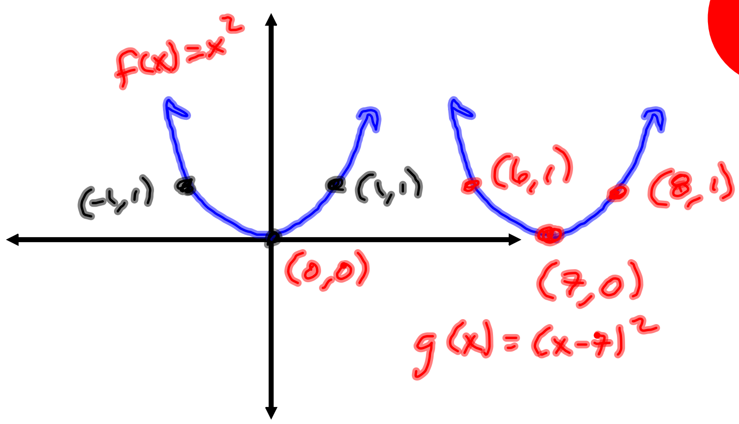
$y = f(x - h), \quad h > 0$ Right shift

$g(x) = (x-7)^2$

$f(x) = x^2$

$g(x) = f(x-7)$

Delay by 7.

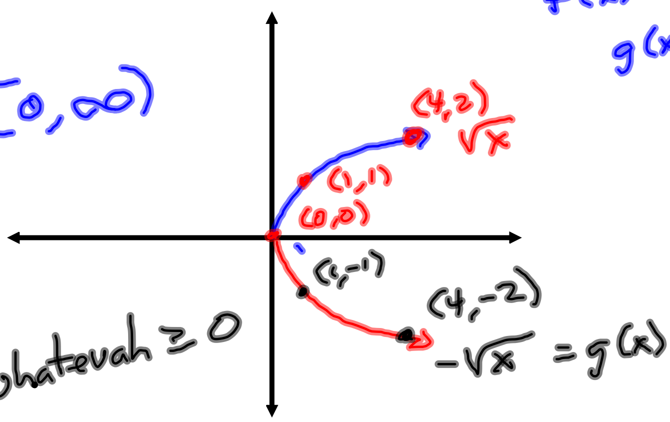


Vertical Flip $\boxed{-f(x)}$

$g(x) = -\sqrt{x}$

$f(x) = \sqrt{x} \Rightarrow g(x) = -\sqrt{x}$

$\mathcal{D}(\sqrt{x}) = [0, \infty)$

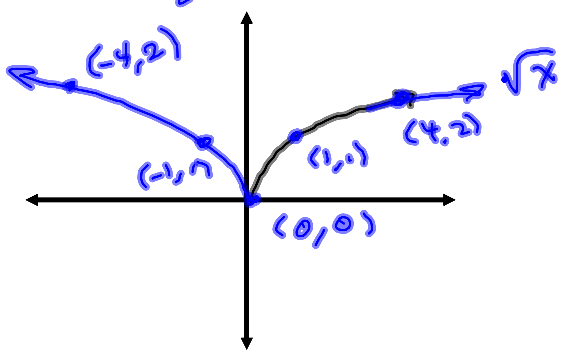


$\sqrt{\text{whatevah}}$

Needs whatevah ≥ 0

Horizontal Flip: $f(-x)$

$g(x) = \sqrt{-x}$



$\mathcal{D}(\sqrt{-x}) = (-\infty, 0]$

Need $-x \geq 0$

$\Rightarrow x \leq 0$

$\Rightarrow \mathcal{D} = \{x \mid x \leq 0\} = (-\infty, 0]$

$$g(x) = -(x+2)^2 + 11$$

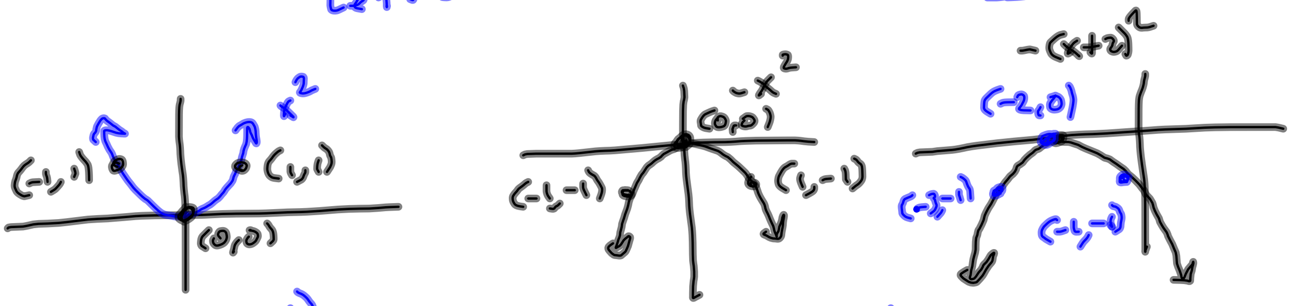
$+11 \to$ save for last

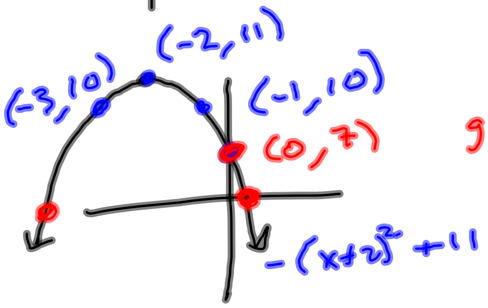
$f(x) = x^2 \xrightarrow{} -x^2 \xrightarrow{} -(x+2)^2 \xrightarrow{} -(x+2)^2 + 11$

Flip ↕ Left 2 up 11

$f(x) = x^2 \xrightarrow{} (x+2)^2 \xrightarrow{} -(x+2)^2 \xrightarrow{} -(x+2)^2 + 11$

Left 2 Flip ↕ up 11





$g(0) = -(0+2)^2 + 11 = -(4) + 11 = 7$ ✓

x-int: Need $-(x+2)^2 + 11 = 0$

$-(x+2)^2 = -11$

$(x+2)^2 = 11$

$|x+2| = \sqrt{11}$

$x+2 = \pm\sqrt{11}$

$x = -2 \pm \sqrt{11}$

$\sqrt{3^2} = 3$

$\sqrt{(-3)^2} = \sqrt{9} = 3$

$\sqrt{x^2} = |x|$

$\boxed{\text{Solving by Square Root Property}}$

Let $f(x) = 2x^2 - x - 1$

when does $f(x) = -1$?

$2x^2 - x - 1 = -1$

$2x^2 - x = 0$

$x(2x-1) = 0$

$x = 0$ or $2x - 1 = 0 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$

$\Rightarrow x \in \{0, \frac{1}{2}\} \rightsquigarrow (0,-1)$ & $(\frac{1}{2}, -1)$ on graph.

Find zeros of $f(x)$:

$2x^2 - x - 1 = 0$

$(2x+1)(x-1) = 0$

$\Rightarrow \dots \Rightarrow x \in \{-\frac{1}{2}, 1\}$

Zeros are $x = -\frac{1}{2}, 1$.

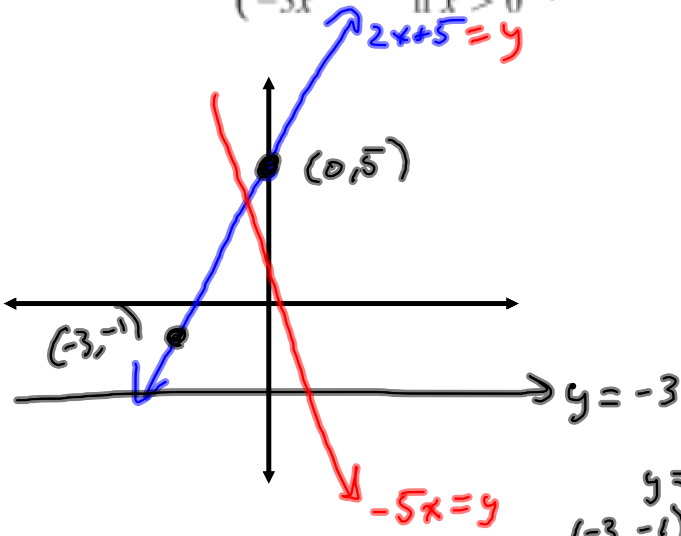
$ax^2 + bx + c = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Back to 1.4 for piecewise-defined functions:

34. $f(x) = \begin{cases} 2x + 5 & \text{if } -3 \le x < 0 \\ -3 & \text{if } x = 0 \\ -5x & \text{if } x > 0 \end{cases}$

Graph.
Domain, Range
from Graph



$D = \{ x \mid -3 \le x < 0 \text{ OR } x = 0 \text{ OR } x > 0 \}$

$= \{ x \mid -3 \le x \}$

$= [-3, \infty)$

Look @ Endpoints:

$y = 2x+5$: $-3 \le x < 0$

(-3,-1) ← $x = -3 \rightsquigarrow 2(-3)+5 = -6+5 = -1$

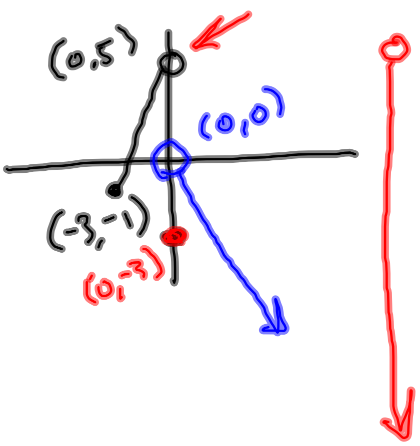
$x = 0 \rightsquigarrow 2(0)+5 = 5$ (0,5)

$y = -3$: $x = 0$

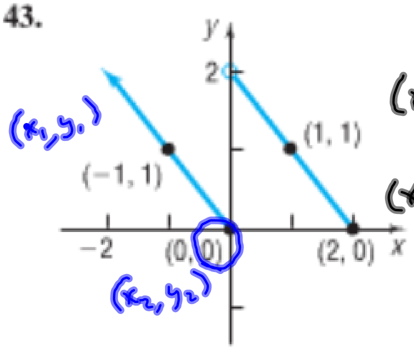
$y = -5x$: $x > 0$

$-5(0) = 0 \rightsquigarrow (0,0)$

$R = (-\infty, 5)$



43.



$$f(x) = \begin{cases} -x & x \le 0 \\ -x+2 & x > 0 \end{cases}$$

$x \le 0$:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-1}{0-(-1)} = \frac{-1}{+1} = -1 = m$$

$$y = mx + b$$
$$y = -1x + 0 = -x$$

$$y - y_1 = m(x - x_1)$$
$$y - 1 = -1(x+1)$$
$$\boxed{y = -(x+1) + 1}$$
$$\boxed{y = m(x - x_1) + y_1}$$

$x > 0$:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-1}{2-1} = \frac{-1}{1} = -1$$

$$y = mx + b$$
$$y = -x + 2$$